Merrymon

A Precision Determination of the Acceleration of Gravity with a Discussion of Corrections

A PRECISION DETERMINATION OF THE ACCELERATION OF GRAVITY WITH A DISCUSSION OF CORRECTIONS

BY

WILLIAM WALTER MERRYMON

A. B. University of Missouri, 1912.

THESIS

Submitted in Partial Fulfillment of the Requirements for the

Degree of

MASTER OF ARTS

IN PHYSICS

IN

THE GRADUATE SCHOOL

OF THE

UNIVERSITY OF ILLINOIS

1917

UNIVERSITY OF ILLINOIS

THE GRADUATE SCHOOL

June 1 1917

I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY WILLIAM WALTER MERRYMON

ENTITLED A PRECISION DETERMINATION OF THE ACCELERATION OF GRAVITY WITH A DISCUSSION OF CORRECTIONS

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF ARTS

A. P. Carman
In Charge of Thesis

A. P. Carman
Head of Department

Recommendation concurred in:*

Committee on Final Examination*

*Required for doctor's degree but not for master's.

T A B L E   O F   C O N T E N T S

I GENERAL CONSIDERATIONS

The object of this work is the accurate determination of "gravity", which is the term used for the phenomena of weight or of the acceleration of a body falling freely to the earth. At any place it is the resultant of the earth's attractive force, "gravitation", and the centrifugal force due to the earth's rotation, and is usually denoted by "g". The acceleration due to unit mass at unit distance is general, and is known as the "gravitation constant"; it is denoted by G.[1] It is thus seen that g is the measure of a particular case of gravitation. The value of g varies a few tenths percent over the surface of the earth, due to change in latitude, and the earth's structure underlying the station. In this work the value of g is to be determined in Room 116 of the Laboratory of Physics of the University of Illinois. This is an inside room on the first floor of the Laboratory. The walls are very heavy, which would minimize the possibility of any jar, and it was designed throughout to be a constant temperature room.

An accurate knowledge of g is highly important from a scientific standpoint, for it enters as a factor in a great many formulas, particularly those in mechanics and allied subjects; for instance, it is essential in determining the mechanical equivalent of heat, in using the disc electrometer, and the electrostatic and high potential voltmeters. A knowledge of both g and G make it possible to compute the mass and density of the earth itself, as well as its compression or shape. The observations of the varying value of g over the earth, and its study, have yielded much of our present knowledge of the shape of the earth.

---

1 Poynting and Thomson, "Properties of Matter", p 7, et seq.

The variations in the time keeping of pendulum clocks when transported about the earth, first led to the discovery, in 1672, of variations of gravity. The swinging pendulum, then, being influenced in its motion by gravity is, and has been thus far, the most convenient and precise instrument with which to study this acceleration. It is now known that gravity varies with latitude, elevation, and the character of the earth's topography.

Galileo, about A.D. 1600 established the laws of the simple pendulum, and recognized its use as a means of determining time intervals. Soon after Mersenne determined the length of the seconds pendulum, and in 1673 Huygens solved the problem of determining the length of a simple pendulum equivalent to a given compound pendulum, and worked out the theory of the latter.

Newton made great use of the pendulum in proving that mass is proportional to weight, and establishing his law of gravitation - the inverse square law. Bouguer, Borda, Bessel, Cassini, and others extended the knowledge of the pendulum and factors which influence its motion. Bouguer[1] in the early part of the 18th century in his scientific expedition to Peru discovered that gravity did not decrease in accordance with the inverse square law at stations on a mountain. He worked out the first formula to give the change in gravity with elevation on land. At present Helmert's formula and the formula deduced by the United States Coast and Geodetic Survey[2] are the most accurate and serviceable.

For the work in hand we are particularly interested in Capt. Kater of the British Navy, who in 1817 devised the form of compound

---

1 Poynting & Thomson, "Properties of Matter", p 23.

2 "Investigation of Gravity and Isostasy" by William Bowie, U.S. C. & G.S. Spec. Pub. No.40, 1917, p 134.

reversible pendulum which has been used a great deal ever since, in theoretical and practical scientific work.

All advanced governments now carry on gravity determination work, and in the United States this is done by the Coast and Geodetic Survey. This bureau has made very accurate determinations of gravity at more than two hundred stations in the United States and has carried out with it investigations of gravity and isostasy[1]. Gravity observations are an important and essential part of a geodetic survey.

---

1 U.S. C.& G. Survey Special Pubs. on "Figure of the Earth and Isostasy" 1909-1910.

"Topog. Effects & Isostatic Compensation", Spec. Pubs. No.10, and No.12, 1912.

"Investigations of Gravity and Isostasy" by Wm. Bowie, U.S. C.&G. Survey, Spec. Pub., No.40, 1917.

II METHODS

In making a determination of g either of two general methods may be followed. Since the period of a pendulum varies as the square root of its length divided by g (See equations No. 1 and 12 below), if both the time of vibration and the length are accurately known in absolute units of any system, the value of the acceleration of gravity may be calculated in absolute units of the same system.

In the practical work of determining g at different stations it is a difficult and tedious task to follow this absolute method. However, it is relatively easy to determine the variation of gravity between stations by determining the variation in the period of a given pendulum at the different stations. If one of these stations is now standardized by an absolute determination of gravity, the absolute value of g at the others may be readily computed. This is known as a "relative method."

The determinations of the United States Coast and Geodetic Survey are of this character. Their standard station is a certain pier in the basement of the Coast and Geodetic Survey Office in Washington D.C. The absolute value of gravity was determined there many times, and in many ways, and in 1900 the relative value of gravity at this pier and the standard pier at Potsdam, Germany, was very accurately determined. The value of g at this pier as determined on the basis of the Potsdam value was adopted as the standard for this country, thus connecting the determinations of the two countries. This has great advantage from a scientific and geodetic standpoint. The value of g thus adopted for this pier is 980.112 dynes or cm. per second per second.

In making determinations of gravity the United States Coast

and Geodetic Survey[1] employ a very carefully made and delicately adjusted set of pendulums, each about one-quarter of a meter long, hence having a half period (or beat) of about one-half second.[2] These are swung at different stations in a special air-tight case, at constant temperature, in an air pressure of 6 cms. Special precautions are taken to insure greatest accuracy. The flexure of the case is observed with an interferometer and corrected for. Their observations have a probable error of only .005 of a dyne.

In the present work some thought was taken of using the ring pendulum method, as described by Dr. Mendenhall.[3] The thought was to use a ring disk pendulum made of glass which almost completely eliminates errors due to non-homogeneity, which can creep in in metals. However, it was found impossible to secure a suitable glass disk finished to the desired degree of accuracy.

This method has several advantages:

1. A definite and easily observable length to measure the external diameter of the ring.
2. The ring disk has great rigidity, hence but slight departure from its measured figure.
3. By swinging from different points, errors of non-homogeneity could be detected and partially corrected for.

The Repsold Pendulum[4] was also considered but time did not allow it to be constructed and used. Its symmetrical construction largely eliminates the air effect in swinging.

---

1 U.S. C.& G. Survey, Spec. Pub. No.23, 1915.

3 Memoirs, Natl. Acad. of Sciences, Vol.X, 1st Memoir.

2 Von Sterneck's Pendulum , Poynting & Thomson, "Properties of Matter", p 25.

4 Poynting & Thomson, "Properties of Matter" p 18.

It was finally decided to use a form of Kater's reversible pendulum, and a very excellent one was available.

III THEORY

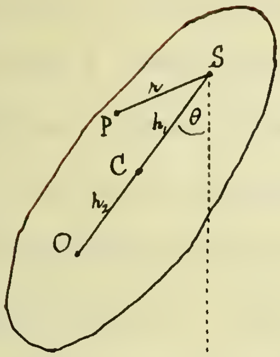

Fig. 1

It is shown in the mechanics of physics[1] that so long as the arc over which a simple pendulum swings is very small, the time of one complete vibration is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad (1)$$

where l is the length of the pendulum and g is the acceleration of gravity.

As stated above, Huygens first developed the theory of the compound pendulum, and Bessel further elaborated it and its applications.

Suppose there is given a rigid body (Fig. 1), which is capable of rotation about a horizontal axis S. This constitutes a compound pendulum.

Let r be the distance of a given point P of the body from the axis of suspension through S, and let w be the angular velocity with which the body is moving.[2]

The linear velocity of Point P is wr

Therefore $$v = wr$$

then the linear acceleration of

$$P = \frac{dv}{dt} = (r\frac{dw}{dt}).$$

If m is the mass of P, the force acting on P at right angles to r is,

$$F = m(r\frac{dw}{dt})$$

and the moment of this force about the axis through S is,

$$mr^2 \frac{dw}{dt} .$$

Likewise for the whole body the moment of the force about S is,

---

1 Duff's Physics, p 88; and Jean's Mechanics, p 259.

2 Watson's Practical Physics, p 120.

$$\frac{dw}{dt} \sum mr^2 \tag{2}$$

The only force acting on the body is the attraction of gravity. For the whole body the total moment of the force of gravity about the vertical plane through S is $Mgh_1 \sin \theta$, where M is the whole mass of the body, supposed concentrated at its center of gravity C, and SC = h, and θ is the angle through which the body has turned from equilibrium. This moment of force is negative when θ is positive, tending to bring the pendulum back to equilibrium position.

Hence

$$\frac{dw}{dt} \sum mr^2 = -Mgh_1 \sin \theta \tag{3}$$

Now

$$w = \frac{d\theta}{dt} \qquad \text{and} \qquad \frac{dw}{dt} = \frac{d^2\theta}{dt^2} = a$$

or the angular acceleration, and $\sum mr^2$ = the moment of inertia, I, of the body about the axis S.

Therefore $$-Mgh_1 \sin \theta = Ia.$$

When θ is small, $\sin \theta$ may be taken equal to θ, in radians, then

$$a = -\frac{Mgh_1}{I} \theta \tag{4}$$

This satisfies the condition for angular harmonic motion and the period of vibration[1] is

$$T_1 = 2\pi\sqrt{-\frac{\theta}{a}} = 2\pi\sqrt{\frac{I}{Mgh_1}} \tag{5}$$

Now let the body be rotated about the center of oscillation O, and let OC = h_2.

As above the period is found to be

$$T_2 = \frac{2\pi\sqrt{I}}{\sqrt{Mgh_2}} \tag{6}$$

---

1 Duff's Physics, p 91.

Now let I_0 be the moment of inertia of such a body about its center of gravity, then from mechanics

$$I = I_0 + Mh_1^2$$

Substituting in (5) above

$$T_1 = \frac{2\pi\sqrt{I_0+Mh_1^2}}{\sqrt{Mgh_1}}$$

But $I_0 = MR^2$ where R is the radius of gyration, therefore,

$$T_1 = 2\pi\sqrt{\frac{R^2+h_1^2}{gh_1}} \tag{7}$$

If this be compared with the formula for a simple pendulum, it is seen that, if l_1 be the length of a simple pendulum which vibrates in the same time as the compound pendulum, then

$$l_1 = \frac{R^2+h_1^2}{h_1} = \frac{R^2}{h_1} + h_1 \tag{8}$$

also $$l_1 - h_1 = h_2$$

Now let O in Fig. 1 be located so that $SO = l_2$. Then when the body is suspended from O the time of vibration from equation (6) is

$$T_2 = \frac{2\pi\sqrt{I}}{\sqrt{Mgh_2}} = \frac{2\pi\sqrt{R^2+h_2^2}}{\sqrt{gh_2}} = \frac{2\pi\sqrt{l_2}}{\sqrt{g}} \tag{9}$$

whence

$$l_2 = \frac{R^2}{h_2}+h_2 \tag{10}$$

From equation (8) $$\frac{R^2}{h_1} = l_1 - h_1 = h_2$$

Substitute these in equation (10)

$$l_2 = \frac{R^2 h_1}{R^2} + l_1 - h_1$$

or

$$l_2 = l_1.$$

This shows that the length of the equivalent simple pendulum is the same, whether suspended from S or O. This leads to the conclusion that the center of suspension and center of oscillation are interchangeable, and the distance between them is the length of the

equivalent simple pendulum.

In the Kater's form of pendulum, the adjustable weights are to be shifted until the period of the pendulum is the same from each knife-edge. It would be necessary for the periods to be exactly equal in order to use the formula (9) above.

In his work with the pendulum in the early part of the 19th century, Bessel showed[1] that it was not necessary to have these periods absolutely equal.

From equations (7) and (9)

$$T_1 = 2\pi\sqrt{\frac{R^2+h_1^2}{gh_1}}$$

$$T_2 = 2\pi\sqrt{\frac{R^2+h_2^2}{gh_2}}$$

squaring and multiplying across,

$$h_1gT_1^2 = 4\pi^2(R^2+h_1^2)$$

$$h_2gT_2^2 = 4\pi^2(R^2+h_2^2)$$

eliminating R^2 by means of these two equations,

$$g(h_1T_1^2-h_2T_2^2) = 4\pi^2(h_1^2-h_2^2)$$

$$\frac{4\pi^2}{g} = \frac{h_1T_1^2 - h_2T_2^2}{h_1^2 - h_2^2}$$

or

$$\frac{4\pi^2}{g} = \frac{T_1^2+T_2^2}{2(h_1+h_2)} + \frac{T_1^2-T_2^2}{2(h_1-h_2)} \qquad (11)$$

or taking t as the time of one oscillation, equals one half complete vibration,

$$\frac{\pi^2}{g} = \frac{t_1^2+t_2^2}{2(h_1+h_2)} + \frac{t_1^2-t_2^2}{2(h_1-h_2)} \qquad (12)$$

These are the formulas to be used in calculating "g" with the Kater's, or reversible pendulum. The distance h_1 is from the center

---

1 Poynting & Thomson, "Properties of Matter", p 15.

of gravity of the pendulum to the knife edge about which the period is t_1; i.e., to the plain end in the following data.

The periods are made very nearly equal, hence the last term in (12) is quite small compared with the first. Therefore h_1 and h_2 individually need to be only approximately known. The position of the center of gravity is determined with sufficient accuracy by balancing the pendulum; and the distance h_1 and h_2 may be measured with a good meter stick. The quantity (h_1+h_2), or the distance between the knife edges, and the times of oscillation t_1 and t_2 must be determined with every care.

Kater Pendulum

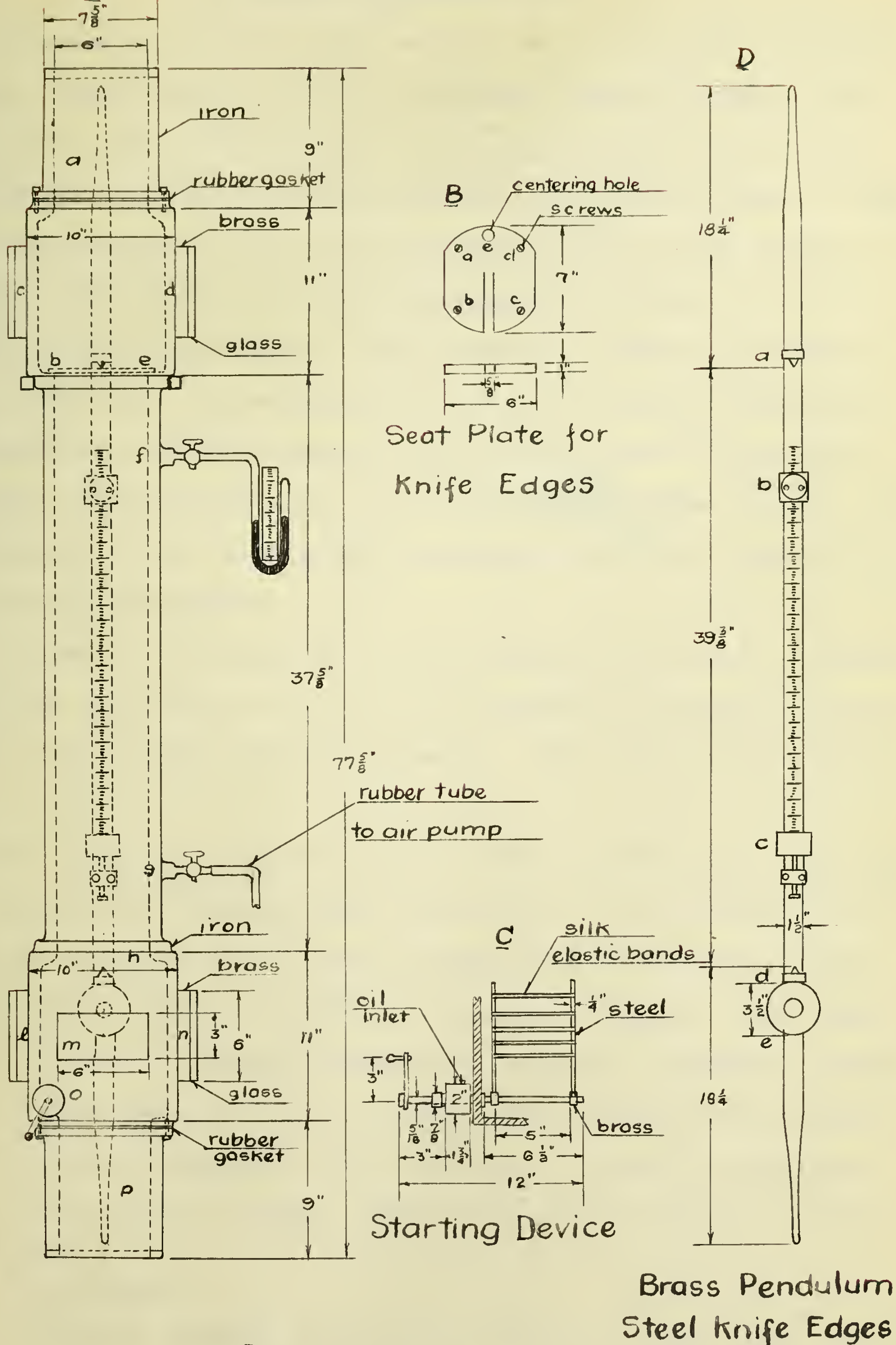

Plate I

IV APPARATUS

The Kater's pendulum used[1] was one made by Max Kohl, Chemnitz, Germany, (See Plate ID). It is a strip or blade of brass 1 1/2" wide, 3/16" thick and 76" long. At equal distances from the two ends the two knife-edges, facing inward, are fixed as shown. They are 1 1/2" in length and are set at right angles to the blade of the pendulum. Next to one of the knife-edges and outside of it, is fixed a 3 1/2" brass weight. This throws the center of gravity nearer this end.[2] Two adjustable weights are placed between the knife edges so that the center of oscillation may be adjusted to fall below either knife-edge just the distance between them. In this condition it is a reversible pendulum having the same period from either knife-edge.

In order to eliminate as far as possible the effect of buoyancy of the air, and flow of air with the pendulum in swinging, it was swung in an air-tight vertical receiver. (See Plate IA and Plate II).

This receiver was made from a 6" iron pipe 39" long. Each end was fitted with a flanging cast iron collar. An 11" piece of 10" iron pipe, called a "head", was screwed to each of these collars, and the ends of these 10" pipes were again fitted with cast iron collars, with a 6" opening, ground and faced for a gasket. Another ground disk fitted over each of these end openings,and a 9" piece of 6" pipe was screwed into it. The ends of these 6" pipes were closed with cast iron plates. Each of these end pieces, or caps,

---

1 PL No.2340

2 Watson's Textbook of Practical Physics, p 127, 128.

above the ground joint was shaped much like a stove pipe hat. They were provided with a slot on opposite sides in the flange on the disk, and bolts were set into the end cast iron collars so that these caps could be drawn tightly down upon the gaskets.

Two 6" square windows were provided in the middle of each of the heads, one window on one side and the other on the opposite side. In addition, a 3"x6" similar window was placed between the two others on the lower head, as shown. The seats for these windows were made of 1/2" brass pieces carefully joined and fitted and soldered to the head around the window openings. The plate glass windows used were 1" thick. Bolts 4" long were set into the receiver head at each corner of the windows and clamps and nuts reached over the edge of the glass to give a pressure upon it when seating the glass window in wax.

This receiver was set in a very heavy cast iron main bracket, which was fastened to a heavy oak board. This board was fastened rigidly, by eight large expansion bolts, to the wall, near the corner of the room. Another cast iron bracket was fastened to this board, below the bottom cap of the receiver, and a bolt set into the cap so that vertical tension could be applied to the receiver to help hold it rigidly. (See Plate II)

In addition cross braces of steel were fastened from the main bracket to smaller oak blocks on the two corner walls. This was to prevent any possible horizontal motion of the receiver.

The threaded joints of the receiver were very thoroughly leaded, and the iron part covered with two coats of iron filler and two coats of enamel paint. The rubber gaskets were set in vaseline. The circular plate for the knife-edges of the pendulum to rest on (See

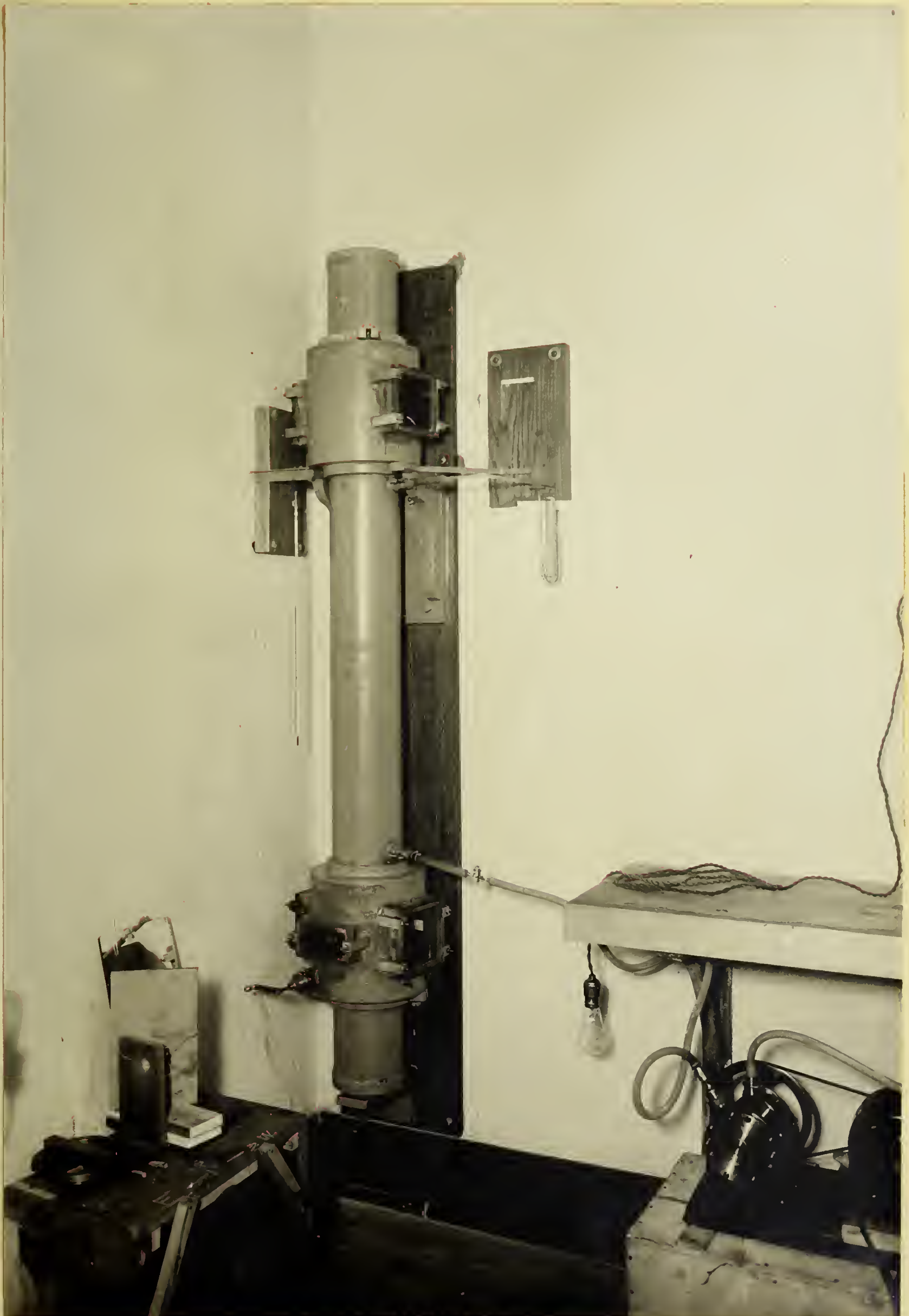

Plate II

Plate IB), was flange fitted to the inside of the upper collar. A 5/8" slot was cut in it from one side, through which the pendulum hung. This plate was centered by a pin and hole and fastened down by four screws. It was set so that the pendulum should swing parallel to the small side window.

The cathetometer[1] used to measure the distance between the knife-edges was one made by the Societe Genevoise. (See Plate III). It was provided with two micrometer eye-piece telescopes. A standard meter of invar steel was used with this cathetometer. The cathetometer, standard meter, and pendulum when measured were mounted as shown in Plate III.

The clock was a Riefler Standard Observatory clock (No.183), mounted in a small room just 20 feet from the receiver. The motor driven air-pump[2] used to exhaust the receiver was of the eccentric pattern made by E. Leybold.

The apparatus used for starting the pendulum, when in the receiver, is shown in Plate IC. It consists of a 5/16" steel rod 12" long. This passes through a brass collar in the lower part of the lower head of the receiver, at the side next to the corner of the room. It is set horizontal and at right angles to the plane in which the pendulum swings. On the outside of the receiver it passes through a 2" cylindrical cup which is soldered to the receiver all around. Inside this cylindrical cup there is a packing cup surrounding the rod. The outer end of the cylindrical cup is closed by a brass disk threaded in, with another packing cup on the outside attached to it. This cylindrical cup is filled with heavy oil thru

---

1 PL No.3271

2 PL No.3576

a hole in the top and the packing cups keep the oil or air from leaking in or out. No leaking or trouble of any kind was experienced with this starting apparatus.

A handle is attached to the rod on the outside, and two parallel upright 8" rods or arms are attached to it on the inside, and set 5" apart. Silk elastic bands are stretched across these arms. When the arms are turned forward the bands push and hold the pendulum to one side, and when it is quiet and the arms are suddenly thrown back the pendulum starts swinging perfectly steady.

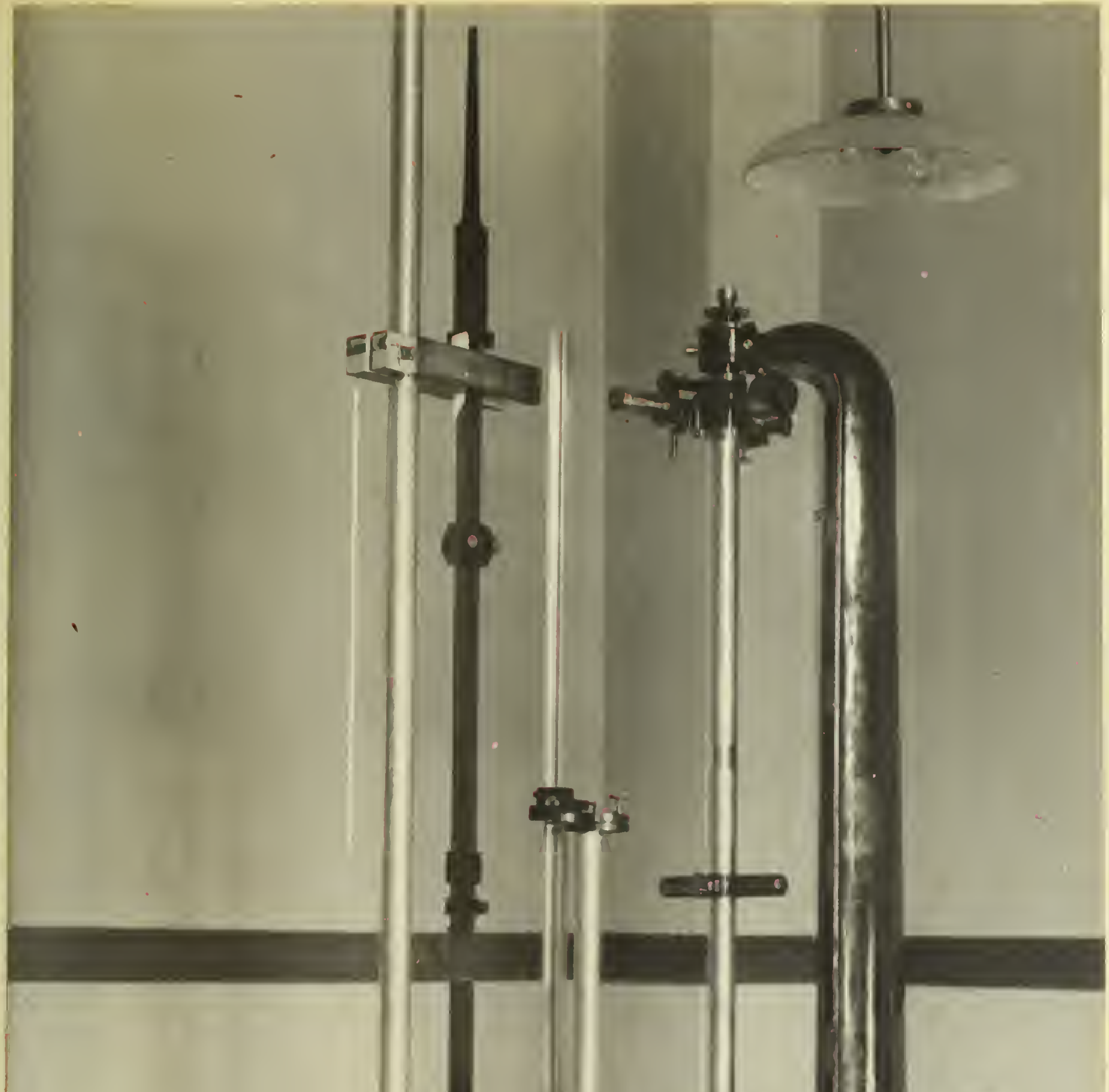

Plate III

V MEASUREMENTS

Measurement of the Distance Between the Knife-edges.- For this work the above mentioned cathetometer (See Plate III), which is provided with two telescopes, an upper and a lower one, was used. It was necessary, in adjusting it, to make the axis of the cathetometer vertical, and the axis of the telescope horizontal, and to focalize the telescopes.[1]

In this form of cathetometer the axis is pivoted at the top and bottom so that the telescopes can first be set upon the knife-edges, and then the axis turned so that they point on the standard scale. The telescopes are each provided with micrometer eye-pieces and the usual movable double wire.

It is necessary to have the object to be measured (the pendulum), and the standard scale at the same distance from the axis of the cathetometer; otherwise they would not both be in focus, and the value in centimeters of one division of the micrometer would not be the same for the two. They were therefore carefully placed at equal distances from the axis by measuring. It was also necessary that both should be vertical. This was accomplished in the case of the scale by a plumb bob. The pendulum hung vertical itself, as the outer ends of its upper knife edge rested on brass plates on the two sides of a U-shaped vertical notch cut in the block supporting it.

The value in centimeters on the standard scale (at the distance employed), of a movement up or down of the horizontal wire of the micrometer, in terms of the divisions on its recording disk, is a constant k, that was determined for each telescope. This was done

---

1 Ferry & Jones, "Practical Physics" p 23; Watson's "Practical Physics" p 58.

by sighting on the standard scale and recording the number of micrometer divisions turned through when the movable wire in the eye-piece appeared to pass from one millimeter on the scale to the next.

Having all the adjustments made, the telescopes were first turned on the pendulum and the movable wire brought to the sharp edge of the knife-edge. The recording disk on the micrometer was now read. Then the telescope was turned toward the scale and the movable wire brought to a millimeter division on the scale. The recording disk was again read. The difference in the two readings multiplied by k was the distance in centimeters at which the knife-edge stood above or below this millimeter on the scale. The position of the other knife-edge was found likewise on the standard scale.

The temperature was read before and after the observations and was kept constant. Every care was taken not to jar the apparatus. The adjustment of the light to a satisfactory position was a source of difficulty, since it must not change the temperature of the pendulum any. The use of paper backgrounds for the knife-edges aided.

The invar standard scale was correct at 20°C., and since the coefficient of expansion of invar is very small, a few degrees change could be entirely neglected.

In the eye-pieces the upper knife-edge stood just above the 999th millimeter, and the lower knife-edge just below the zero of the standard scale. Hence both fractions measured by the micrometer are to be added to the 999 millimeters to obtain the length of the pendulum between the knife-edges. The micrometer was provided with the usual two movable wires. These were each used in succession, and two readings made with each on the part of the knife-edge on

either side of the blade of the pendulum. In using one wire it would be run to one millimeter division on the standard scale, and the other wire would be run to the other of the two scale divisions between which the knife-edge stood. (See Tables II and III)

The pendulum was balanced on a knife-edge and the distances, h_1 and h_2 (See equation (12) above), from the center of gravity to each knife edge, were measured with a meter stick. (See Table I)

TABLE I

From center of gravity	To loaded end h_2	To plain end h_1
(h_1-h_2) = 14.58 cm.	42.75 cm.	57.25 cm.
$2(h_1-h_2)$ = 29.16 cm.	42.65 cm.	57.30 cm.
	h_2 = 42.70 cm.	h_1 = 57.28 cm.

TABLE II

Temperature 22°.3C   Upper Micrometer

Upper Wire Trial	Microm. reading on knife-edge (Notches)	Microm. reading on 999th mm. (Notches)	Difference (d) (Notches)	Difference (e) (mm.)	Difference to 999th mm.
1	29.025	29.665	.640	.192	.192
2	28.910	29.680	.770	.231	.231
3	28.985	29.695	.710	.213	.213
4	29.055	29.695	.640	.192	.192
Lower Wire		**Microm. reading on 1000th mm. of scale**			**(1-e)**
1	29.425	26.685	2.740	.822	.178
2	29.365	26.685	2.680	.804	.196
3	29.400	26.690	2.710	.813	.187
4	29.415	26.69	2.725	.816	.184

Upper knife-edge above 999th mm. of scale,   Mean .197 mm.

Upper knife-edge position = 999.197 mm. on scale

k_1 for Upper Micrometer:

1 Notch (in eye-piece) = 100 divisions on recording disk

= .3003 mm.

k_2 for Lower Micrometer:

1 Notch = .3096 mm.

TABLE III

Temperature 22°.1C Lower Micrometer

Upper Wire Trial	Microm. reading on knife-edge (Notches)	Microm. reading on zero mm. of scale (Notches)	Difference (d) (Notches)	Difference (e) (mm.)	Difference to zero of scale mm.
1	25.310	24.120	1.190	.358	.358
2	25.255	24.130	1.125	.337	.337
3	25.270	24.120	1.150	.346	.346
4	25.290	24.115	1.175	.352	.352
Lower Wire		Microm. reading on 1st mm. (Notches	(d)	(e)	(1-e)
1	25.580	27.665	2.085	.628	.372
2	25.520	27.665	2.145	.644	.356
3	25.535	27.680	2.145	.644	.356
4	25.630	27.660	2.03	.612	.388

Lower knife-edge below zero of scale, Mean = .358 mm.

Upper knife-edge position on scale (Table II) 999.197 mm.

Length of pendulum at 22°.2C = 999.555 mm.

Coefficient of expansion of brass = .187 x 10^{-4} per 1°C.

Length of pendulum at 24°.3C = (h_1+h_2) = 99.9594 cm.

$2(h_1+h_2)$ = 199.9188 cm.

Determination of Period.- The pendulum was first suspended from a steel bracket in another room and its period obtained about each knife edge. This was obtained from the mean of ten groups of 300 transits each[1], taking the time of the 0th, 25th, 50th, etc., to the 225th, and then next the 300th, etc., to the 525th. Subtracting the 0th from the 300th gives the time of one group, likewise the time of the 25th from that of the 325th gives another, and so on.

The movable weights were shifted until t_1 (loaded end down) was determined as 1.00142 seconds, and t_2 (loaded end up) was determined as 1.00130 seconds.

The Riefler Standard Observatory clock was regulated, during the previous months, by the wireless time signals sent from the Arlington, Va. Station, at eleven o'clock Central Standard Time each day. The rate of the clock was only a few hundredths of a second per day, during the course of the observations, and therefore no correction was required to the observed time intervals.

As previously mentioned, the room in which the receiver was mounted was an inner room in which the temperature could be kept constant to within a few tenths of a degree. The mean of the temperatures during observation was 24°.3C. The length of the pendulum as determined by the cathetometer at 22.2°C. was reduced to this temperature by using the coefficient of expansion of brass, and no further correction to its length was required. The door was opened and an electric fan was operated occasionally to bring down the temperature when it tended to rise. Most of the observations were made at night, when this was most easily accomplished.

---

1 Miller's Labor Manual of Physics, p 102.

The precautions taken in the construction of the apparatus to avoid, as far as possible, any flexure or movement of the receiver, have been described above. A small mirror fastened to the main bracket and reflecting a spot of light across the room, indicated no movement of the apparatus.

The air in the receiver was pumped out to as high a vacuum as could be obtained. A stiff wax ("hald-and-half wax") half resin and half bees-wax, heated together over a water bath, was used to cover any places that might leak air. It was put on hot, and by this means a vacuum between 1 and 1.5 cm. was maintained throughout the experiments. There was still a small, slow leak, but it was necessary to operate the air pump only a portion of the time to keep the pressure below the above value. The air-pump itself was set on a box away from the wall and connected to the receiver by a long piece of pressure tubing. A glass valve was connected into this tubing near the receiver, so that the pump could be cut off and the vacuum held. There was no evidence of any jar from the air pump.

The glass windows were set on with the same "half-and-half wax". The window seat as well as the glass must be heated to accomplish this successfully. After placing the glass against the hot wax on the brass seat, a little pressure by the nuts upon the clamps at the corners press the glass down air-tight.

The arc of swing of the pendulum can be observed through the small side window on a mirrored scale placed inside the lower head and at the back side. The distance from the knife-edge above to this scale is known (113.4 cms.). This length together with the reading of the amplitude of the swing on the scale at the beginning and end of each set of observations, enables one to reduce the time

of swing to that of an infinitely small arc.[1]

As the apparatus stands (See Plate II), the pendulum vibrates in a plane parallel to the small side window. The clock is near to the wall on which the receiver is fastened and vibrates in a plane at right angles to that of the pendulum.

For observing the period of the pendulum the method of coincidences was used.[2] Mirrors were placed on a stand near the receiver as shown. The first mirror, against the wall, reflects the image of the clock's pendulum directly. The second mirror reflects an image of the Kater pendulum into the first mirror. By adjusting this second mirror, the image of a small hole in the center of the Kater pendulum may be seen in the first mirror very near the top edge of the image of the second mirror, and just below the image of the lower point of the clock pendulum.

An iron ring stand was used as a rest to steady the head of the observer during observations. The observations were carried out with the eye. A telescope could not be used readily because the images in the mirror were at different distances, yet the eye could follow the two with ease. The times of coincidence were observed for several hours in each instance. The exact time of coincidence could not be determined, for the intervals were very long,and for from 20 to 40 seconds the images appeared to swing together. However, the mean time of the apparent beginning and ending of a coincidence was probably quite close,since the interval was over 1300 seconds in

---

1 Watson's "Practical Physics", p 120.

2 Routh's "Elementary Rigid Dynamics", p 75.

every case.

Between consecutive coincidences the Kater pendulum, which was losing, lost one complete oscillation, or two half-periods or beats. The number of seconds in the interval was therefore divided by the number of seconds less two to obtain this half period, t_1 and t_2.

VI RESULTS AND CALCULATIONS

Two sets of observations in vacuum were made with the pendulum hanging both erect and inverted. One set of observations was also made in each position at atmospheric pressure, as a comparison.

The periods were corrected for amplitude according to the formula[1]: the period with infinitely small arc = observed period x $(1 - \frac{\theta\,\theta'}{16})$ where θ and θ' are the initial and final amplitudes in radians.

VALUES OF "g"

Using equation (12) and the data from the tables:

From Table IV $t_1^2+t_2^2 = 2.004767^s$; $t_1^2-t_2^2 = 0.001217$

From Table III $2(h_1+h_2) = 199.9188$

From Table I $2(h_1-h_2) = 29.16$ cm.

Then by equation (12)

$$\frac{\pi^2}{g} = \frac{2.004767}{199.9188} + \frac{.001217}{29.16}$$

$$= .0100279 + .0000417$$

$$= .0100696. \quad (\pi^2 = 9.869604401)$$

Therefore $g = \frac{\pi^2}{.0100696} = 980.139$ cm. per sec^2.

Computed from C.& G.S. Formula[2] $g = 980.120$ cm. per sec^2.

Computed from Smithsonian Formula[3] $g = 980.114$ cm. per sec^2.

Determined by Mr. Pinkney in 1915[4] $g = \underline{980.099}$ cm. per sec^2.

Mean value of $g = 980.118$ cm. per sec^2.

---

1 Watson's "Practical Physics", p 120.

2 Investigations of Grav. & Isostasy, Wm. Bowie, U.S.C. & G.S. Spec. Pub. No.40, 1917, p 134.

3 Smithsonian Physical Tables, p 104.

4 Determination of "g". Thesis, U. of I. Library, by Mr. L.A. Pinkney, 1915.

TABLE IV

Kater Pendulum Swinging in Vacuum, Heavy End Down

Mean Temperature 24°.4C Set I May 24, 1917

Amplitude of Arc of Swing mins.of arc	Times of Coincidence	Clock Interval	Half-Periods t_1	Mean Half-Period Corrected for Amplitude
43'.9	$20^h 24^m 25^s$	m s	s	
	20 47 10	22 45	1.001466	
	21 09 31	22 21	1.001492	
	21 31 47	22 16	1.001498	
	21 54 00	22 13	1.001502	
	22 15 30	21 30	1.001552	
	22 37 40	22 10	1.001504	
30'.3	22 59 35	21 55	1.001522	
		Mean	$1^s.001505$	$1^s.001500$

Mean Temperature 24°.2C Set II May 25, 1917

			t_1	
48'.5	$20^h 11^m 30^s$	m s	s	
	20 34 00	22 30	1.001482	
	20 56 13	22 13	1.001502	
	21 18 33	22 20	1.001494	
	21 40 42	22 09	1.001506	
38'.8	22 02 55	22 13	1.001502	
		Mean	1.001498	$1^s.001490$

Hence: Mean value of t_1 = $1^s.001495$

and t_1^2 = $1^s.002992$

(From Table V) t_2^2 = $1^s.001775$

$t_1^2+t_2^2 = 2^s.004767$

$t_1^2-t_2^2 = 0^s.001217$

TABLE V

Kater Pendulum Swinging in Vacuum, Heavy End Up

Mean Temperature 24°.2C Set I May 26, 1917

Amplitude of Arc of Swing mins. of arc	Times of Coincidence	Clock Interval	Half-Periods t_2	Mean Half-Period Corrected for Amplitude t_2
36'.4	$2^h54^m00^s$	m s	s	
	3 30 50	36 50	1.000906	
	4 08 00	37 10	1.000897	
	4 45 45	37 45	1.000882	
	5 23 12	37 27	1.000890	
19'.7	6 00 32	37 20	1.000892	
		Mean	$1^s.000893$	$1^s.000889$

Mean Temperature 24°.36C Set II May 26, 1917

				t_2
33'.3	$6^h11^m22^s$	m s	s	
	6 48 32	37 10	1.000898	
	7 26 00	37 28	1.000890	
	8 03 28	37 28	1.000890	
	8 41 10	37 42	1.000884	
	9 18 58	37 48	1.000882	
	9 56 35	37 37	1.000886	
15'.2	10 34 10	37 35	1.000886	
		Mean	1.000888	$1^s.000885$

Mean value of t_2 = $1^s.000887$

t_2^2 = $1^s.001775$

TABLE VI

Kater Pendulum Swinging in Air, Heavy End Down

Mean Temperature 24°.5C May 24, 1917

Amplitude of Arc of Swing mins. of arc	Times of Coincidence	Clock Interval	Half-Periods t_1'	Mean Half-Period Corrected for Amplitude t_1'
57'.6	$16^h 21^m 00^s$	m s	s	
	16 41 20	20 20	1.001640	
	17 01 35	20 15	1.001648	
	17 21 45	20 10	1.001654	
	17 42 00	20 15	1.001648	
	18 02 17	20 17	1.001644	
16'.6	18 22 35	20 18	1.001644	
		Mean t_1' =	$1\overset{s}{.}001646$	$1\overset{s}{.}001641$
		$(t_1')^2$ =		$1\overset{s}{.}003283$

Kater Pendulum Swinging in Air, Heavy End Up

Mean Temperature 24°.0C May 29, 1917

Amplitude of Arc of Swing mins. of arc	Times of Coincidence	Clock Interval	Half-Periods	t_2'
56'.0	$13^h 09^m 45^s$	m s	s	
	13 41 50	32 05	1.001040	
	14 14 15	32 25	1.001029	
	14 46 30	32 15	1.001035	
	15 19 00	32 30	1.001027	
	15 51 30	32 30	1.001027	
7'.6	16 23 50	32 20	1.001032	
		Mean t_2' =	1.001030	$1\overset{s}{.}001030$
		$(t_2')^2$ =		$1\overset{s}{.}002061$

$(t_1')^2+(t_2')^2 = 2\overset{s}{.}005344$

$(t_1')^2-(t_2')^2 = \overset{s}{.}001222$

VII DISCUSSION

The value of gravity thus obtained is in accordance with the three other accurate values stated. It is about .002% different from the mean of the several.

The value of "g" as computed by the C. & G.S. formula has a probable error of .02 dynes, and the value of this determination is within that limit.

The value of "g" determined by equation (12) and the data of Table VI observed with the pendulum swinging in air is

$$g = 979.837 \text{ cms. per sec}^2.$$

This value is .03% different from the mean of the above four values, although every precaution was taken in making the determination.

The air has a buoying effect upon a pendulum reducing its weight; some of the air is dragged along with it, hence virtually increasing its mass, and the air also offers resistance to the pendulum's motion. Consequently at a higher density of air the above factors combine to increase the period, and hence they decrease the value of gravity determined by an observation in air.

The formula for gravity given in the above mentioned U.S. C. & G.S. Spec. Pub. No.40, is

$$g_\phi = 978.039(1+0.005294 \sin^2\phi - 0.000007 \sin^2 2\phi)$$

The first term of the formula is the theoretical value of gravity at the equator, and g_ϕ is the value of gravity at sea level in the latitude of the station. This is very similar to Helmert's formula, the correction for elevation given on page 49 of that publication is used; namely,

$$\text{Elevation correction} = -0.0003086H$$

where H is the elevation in meters. The latitude of the Laboratory is 40° 06' 40" North, and the elevation is 725 ft. = 221 meters. This computed value was also corrected for topography and compensation. This correction was interpolated from the corrections given for surrounding stations on page 50 of the above mentioned publication.

The value of g from the formula given in the Smithsonian Physical Tables was computed from the data given on page 104. This formula is,

$$g_{\phi} = g_{45^\circ}(1 - .002662 \cos 2\phi) \left[1 - \frac{2h}{R}\left(1 - \frac{3}{4}\frac{d}{D}\right)\right]$$

where h is the elevation above sea level, R is the earth's radius, d is the density of the surface strata, and D the mean density of the earth. For ordinary elevations on land $\frac{d}{D}$ is nearly $\frac{1}{2}$. The value of g at 45° lat. is taken from the tables on the same page as 980.6 cm. per sec^2. The elevation correction given on the same page as -.00588 cm. per sec^2. for each 100 ft. of elevation was used.

In a future determination it would be desirable to have agate seats set into the plate for the knife edges to rest on. They leave a slight trace on the iron plate, and possibly cut in very slightly. In this determination, however, every effort was made to avoid any such effect, and the pendulum was set down with the greatest possible ease. Judging by the computed values of g this frictional effect did not enter to a finite extent, for it would have lengthened the period and decreased the value of g.

In Fig.II I have placed the observed values of g taken from the above C. & G.S. publication next to the names of some cities in and near Illinois. Next to Urbana I have placed the value of g determined by these observations. The figures in black are the observed

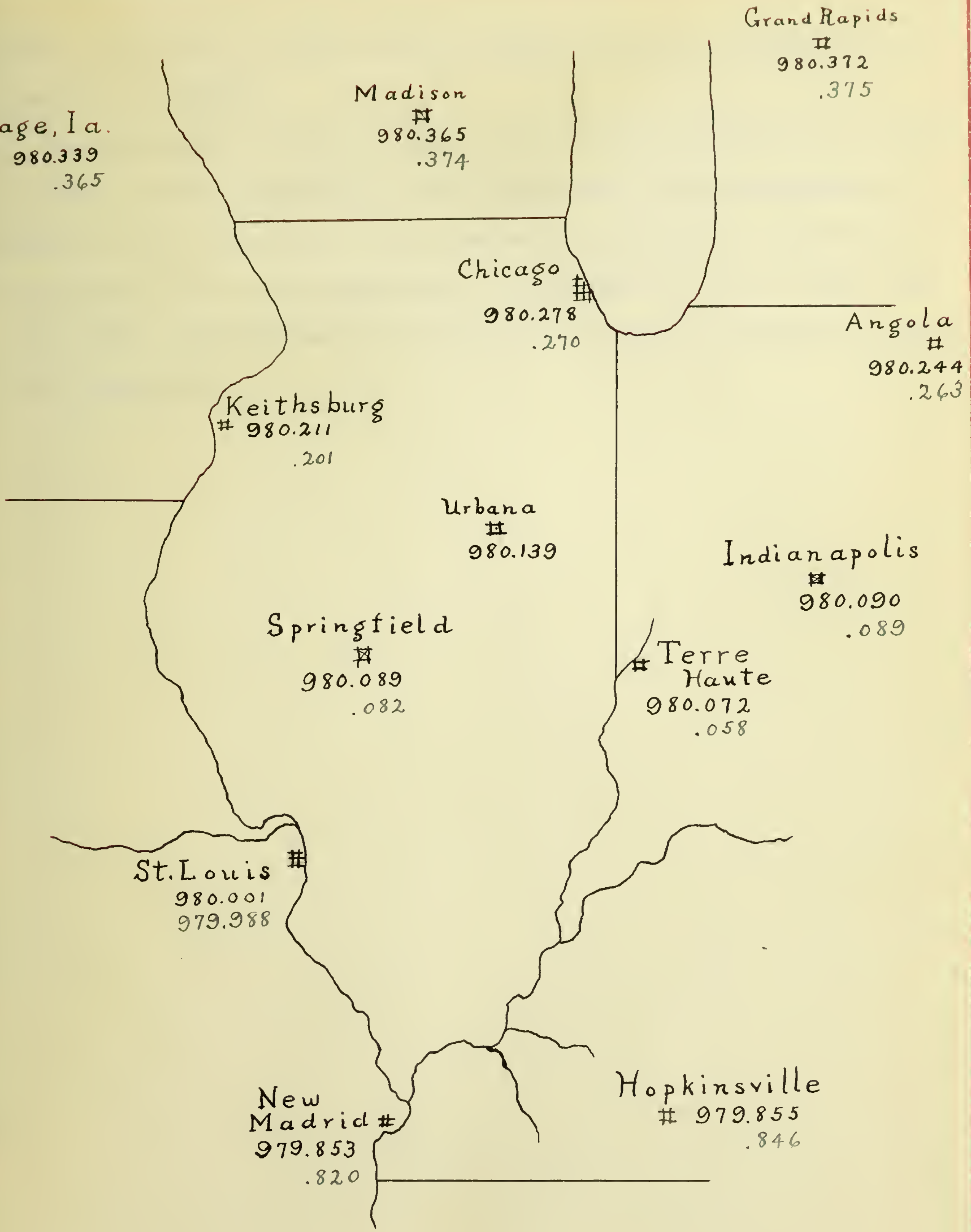

Fig. II Values of the Acceleration of Gravity at Stations in and near Illinois.

values, and those in green ink the observed values reduced to the elevation of Urbana.

In conclusion I wish to express my sincere gratitude to Professor Carman for his assistance and suggestions and encouragement. Thanks are also due almost every member of the Physics Department for valuable help and suggestions in connection with the work, and to Mr. Hays, Mechanician, for his skill and care in making the parts of the apparatus.